Name:

Qualifying Exam Spring 2020
Numerical Analysis Part Two

1. (15 pts) Prove 0-stability of the trapezoidal method.

2. (20 pts) Compute the stability function of the Runge-Kutta method with the following Butcher tableau

$$\begin{array}{c|ccc} 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ \hline & \frac{1}{6} & \frac{4}{6} & \frac{1}{6} \end{array}$$

3. (15 pts) Consider the multistep method

$$11y_{n+3} + 27y_{n+2} - 27y_{n+1} - 11y_n = 3h(f_{n+3} + 9f_{n+2} + 9f_{n+1} + f_n).$$

Is this method consistent? Is this method 0-stable?

4. (25 pts) Consider the shooting method for the BVP

$$y''(t) = y(t)^2 \sin(t + y'(t)), \quad y(0)y'(0) = 1, \quad y(1) + y'(1) = 6.$$

Show how to set up the Newton method and state how to obtain the value of

a) a function whose zero needs to be found

b) the derivative of the function.

5. (25 pts) Consider the BVP

$$u'''' = R(u'u'' - uu'''), \quad u(0) = u'(0) = 0, \quad u(1) = 1,\ u'(1) = 0.$$

Convert it to a system of first order equation $y' = f(y)$. To solve it by the finite difference method using midpoint method one obtains

$$y_k = y_{k-1} + h\, f\left(\frac{y_k + y_{k-1}}{2}\right), \quad k = 1, \ldots, N,\ h = \frac{1}{N}.$$

Let z be an approximation y and assume $w = y - z$ is small. Linearization of f gives

$$w_k = w_{k-1} + \frac{h}{2} A_k(w_k + w_{k-1}) - q_k.$$

Provide an expression for A in terms of z.